

Gateway Algebra I Learner Expectations and Performance Indicators

Standard 1.0: Number and Operations

Students will recognize, represent, model, and apply real numbers and operations verbally, physically, symbolically, and graphically.

Learning Expectations:

The student will:

- 1.1 demonstrate an understanding of the subsets, properties, and operations of the real number system;
- 1.2 demonstrate an understanding of the relative size of rational and irrational numbers;
- 1.3 articulate, model, and apply the concept of inverse (e.g., opposites, reciprocals, and powers and roots);
- 1.4 describe, model, and apply inverse operations;
- 1.5 apply number theory concepts (e.g., primes, factors, divisibility and multiples) in mathematical problem solving;
- 1.6 connect graphical and symbolic representations of absolute value;
- 1.7 use real numbers to represent real-world applications (e.g., slope, rate of change, probability, and proportionality);
- 1.8 use a variety of notations appropriately (e.g. exponential, functional, square root);
- 1.9 select and apply an appropriate method (i.e., mental mathematics, paper and pencil, or technology) for computing with real numbers, and evaluate the reasonableness of results;
- 1.10 perform operations on algebraic expressions and informally justify the procedures chosen;
- 1.11 perform operations on matrices in real-world problem solving (i.e., addition, subtraction, and scalar multiplication).

Performance Indicators State :

As documented through state assessment –

At Level 1, the student is able to

- select the best estimate for the coordinate of a given point on a number line (only rational);
- identify the opposite of a rational number;
- determine the square root of a perfect square less than 169;
- use exponents to simplify a monomial written in expanded form;
- apply order of operations when computing with integers using no more than two sets of grouping symbols and exponents 1 and 2;
- select a reasonable solution for a real-world division problem in which the remainder must be considered.

At Level 2, the student is able to

- order a given set of rational numbers (both fraction and decimal notations);
- identify the reciprocal of a rational number;
- add and subtract algebraic expressions;
- multiply two polynomials with each factor having no more than two terms;
- use estimation to determine a reasonable solution for a tedious arithmetic computation;
- select ratios and proportions to represent real-world problems (e.g. scale drawings, sampling, etc.).

At Level 3, the student is able to

- apply the concept of slope to represent rate of change in a real-world situation.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- connect a variety of real-world situations to integers;
- use manipulatives to represent commutative and associative properties of addition and multiplication;
- investigate alternate algorithms that show the relationship of division to subtraction and multiplication to addition;
- analyze prime and composite numbers;
- compare and contrast the GCF and LCM of a set of numbers;
- refine strategies for estimating whole numbers, fractions, and percentages.

At Level 2, the student is able to

- probe the relationships among various subsets of the real number system;
- compare and contrast the GCF and LCM of a set of algebraic expressions;
- construct a number line to describe the absolute value of a number as distance from zero;
- model operations using real-world situations and physical representations;
- perform operations on matrices using appropriate technology (addition, subtraction, and scalar multiplication);
- explore various representations of absolute value.

At Level 3, the student is able to

- research the history of prime numbers and their uses;
- scrutinize approximate values of real numbers such as pi and the square root of two.

Sample Tasks:

Students design a concept map that illustrates the relationship among decimals, fractions, and percents. Students summarize in writing their concept maps and discuss how equivalent fractions, decimals, and percents can be flexibly interchanged. Students justify the selection of fraction, decimal, or percent notation in specific situations.

Linkages:

Mathematics - Estimation, Measurement, and Computation. Make connections to scientific notation used in science, social studies, and finance.

Standard 2.0: Algebra

Students will describe, extend, analyze, and create a wide variety of patterns and functions using appropriate materials and representations in real world problem solving.

Learning Expectations:

The student will:

- 2.1 recognize, analyze, extend, and create a variety of patterns;
- 2.2 use algebraic thinking to generalize a pattern by expressing the pattern in functional notation;
- 2.3 solve linear systems using a variety of techniques;
- 2.4 communicate the meaning of variables in algebraic expressions, equations, and inequalities;
- 2.5 identify and represent a variety of functions;
- 2.6 apply and interpret rates of change from graphical and numerical data;
- 2.7 analyze graphs to describe the behavior of functions;
- 2.8 interpret results of algebraic procedures;
- 2.9 apply the concept of variable in simplifying algebraic expressions, solving equations, and solving inequalities;
- 2.10 interpret graphs that depict real-world phenomena;
- 2.11 model real-world phenomena using functions and graphs;
- 2.12 articulate and apply algebraic properties in symbolic manipulation;
- 2.13 analyze relationships which can and which cannot be represented by a function;
- 2.14 graph inequalities and interpret graphs of inequalities;

2.15 describe the domain and range of functions and articulate restrictions imposed either by the operations or by the real-life situations which the functions represent;

2.16 describe the transformation of the graph that occurs when coefficients and/or constants of the corresponding linear equations are changed.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- extend a geometric pattern;
- extend a numerical pattern;
- translate a verbal expression into an algebraic expression;
- evaluate a first degree algebraic expression given values for one or more variables;
- solve one- and two-step linear equations using integers (with integral coefficients and constants).

At Level 2, the student is able to

- select the algebraic notation which generalizes the pattern represented by data in a given table;
- translate a verbal sentence into an algebraic equation;
- select the graph that represents a given linear function expressed in slope-intercept form;
- solve multi-step linear equations (more than two steps, variables on only one side of the equation);
- solve multi-step linear equations (more than two steps, with variables on both sides of the equation);
- solve multi-step linear equations (more than two steps, with one set of parentheses on each side of the equation);
- select the linear graphs that models the given real-world situation described in a narrative (no data set given);

- select the linear graph that models the given real-world situation described in a tabular set of data;
- evaluate an algebraic expression given values for one or more variables using grouping symbols and/or exponents less than four;
- determine the slope from the graph of a linear equation (no labeled points);
- apply the concept of rate of change to solve real-world problems;
- select the appropriate graphical representation of a given linear inequality;
- select the non-linear graph that models the given real-world situation or vice versa;
- identify the graphical representation of the solution to a one variable inequality on a number line.

At Level 3, the student is able to

- solve multi-step linear inequalities in real-world situations;
- recognize the graphical transformation that occurs when coefficients and/or constants of the corresponding linear equations are changed;
- determine the domain and/or range of a function represented by the graph of real-world situations;
- select the system of equations that could be used to solve a given real-world problem; *
- find the solution to a quadratic equation given in standard form (integral solutions and a leading coefficient of one); *
- select the solution to a quadratic equation given solutions represented in graphical form (integral solutions and a leading coefficient of one); *
- select one of the factors $(x + 3)$ of a quadratic equation (integral solutions and a leading coefficient of one); *
- select the discriminant of a quadratic equation (integral solutions and a leading coefficient of one). *

* Recommended by the 2003 committee as additional state performance indicators. Additional state performance indicators will begin to be assessed during 2005-2006.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- analyze rational number patterns;
- describe in writing the pattern for real-world data listed in a function table.

At Level 2, the student is able to

- produce an equation to describe the relationship between data sets;
- explore patterns including Pascal's Triangle and a Fibonacci sequence;
- solve a system of two linear equations using the graphing, elimination, and substitution methods;
- defend the selection of a method for solving a system of equations;
- represent algebraic expressions and operations using manipulatives;
- model the steps for solving simple linear equations using manipulatives;
- write an equation that symbolically expresses a problem solving situation;
- justify correct results of algebraic procedures;
- distinguish between a function and other relationships.

At Level 3, the student is able to

- analyze "families of functions" using technology;

Sample Tasks: Use an almanac or the internet to find the area and the average depth of the world's ten largest bodies of salt water. Draw a scatterplot showing the relationship between these two sets of data. Describe the relationship and determine if it is a functional relationship.

Links: Mathematics - Statistics and Probability. Have students recognize the use of patterns in other disciplines and in a variety of cultures

Standard 3.0: Geometry

The student will investigate, model, and apply geometric properties and relationships.

Learning Expectations:

The student will:

3.1 apply geometric properties, formulas, and relationships to solve real-world problems;

3.2 solve problems using the midpoint formula;

3.3 apply right triangle relationships including the Pythagorean Theorem and the distance formula;

3.4 find and represent solutions of quadratic equations geometrically.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- identify ordered pairs in the coordinate plane.

At Level 2, the student is able to

- apply the given Pythagorean Theorem to a real life problem illustrated by a diagram (no radicals in answer);
- apply proportion and the concepts of similar triangles to find the length of a missing side of a triangle.

At Level 3, the student is able to

- calculate the distance between two points given the Pythagorean Theorem and the distance formula.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- describe real-world uses of geometric formulas and relationships
- discuss issues related to estimating areas of irregular-shaped figures for real-world uses (i.e. fencing, painting, laying carpet, purchasing wallpaper or border).

At Level 2, the student is able to

- explain how to determine if a triangle is a right triangle given the measurements of all three sides;
- illustrate the Pythagorean Theorem by measuring the length, width, and diagonals of rectangular objects; design area models to illustrate the Pythagorean Theorem.

At Level 3, the student is able to

- determine the height of an object that is difficult to measure by using the properties of similar triangles.

Sample Task: Approximate the value of pi (π) by looking at the relationship between the diameter and circumference of various circular objects after measuring using a string or a tape measure. Students research and write about how various geometric properties are used in careers such as construction, drafting, and surveying.

Linkages:

Mathematics - Estimation, Measurement, and Computation, Research, and the geometric applications in art.

Standard 4.0: Measurement

The student will apply appropriate tools and units of measurement to produce reasonable results.

Learning Expectations:

The student will:

4.1 use concepts of length, area, and volume to estimate and solve real-world problems;

4.2 apply and communicate measurement concepts and relationships in algebraic and geometric problem-solving situations;

4.3 demonstrate an understanding of rates and other derived and indirect measurements (e.g., velocity, miles per hour, revolutions per minute, cost per unit);

4.1 make decisions about units, scales, and measurement tools that are appropriate for problem situations involving measurement;

4.2 analyze precision, accuracy, tolerance, and approximate error in measurement situations.

Performance Indicators State:

As documented through state assessment –

At Level 1, the student is able to

- estimate the area of irregular geometric figures on a grid;
- calculate rates involving cost per unit to determine the best buy (no more than three samples)'
- apply the given formula to determine the area or perimeter of a rectangle.

At Level 2, the student is able to

- apply the given formula to find the area of a circle, the circumference of a circle, or the volume of a rectangular solid.

At Level 3, the student is able to

- select the area representation for a given product of two one-variable binomials with positive constants and coefficients.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- justify the selection of a unit of measure in specific situations;
- defend estimates of the perimeter and/or area of rectangles and triangles;

- discover and explain formulas used to compute area and volume.

At Level 2, the student is able to

- describe the procedure for determining the area of a composite shape in a real-world situation;
- generalize area formulas using manipulatives for a parallelogram, a triangle, and a trapezoid;
- defend an estimate for the volume of a container;
- relate the volume of a container to its shape;
- analyze precision, accuracy, tolerance, and approximate error in measurement situations.

At Level 3, the student is able to

- discover the dimensions of a rectangle when given its area and the relationship between two adjacent sides;
- describe how changes in the dimensions of figures affect perimeter, area, and volume.

Sample Task: Place students in small groups giving each group a different length of string. Have each group form a rectangle with the string. Ask each group to measure the sides of their rectangle and find its area. Using the string, direct each group to construct the rectangle with the greatest possible area. Give each group the opportunity to justify their solution.

Linkages: Mathematics – Geometry. Use formulas in Science. Discuss connections to drafting and carpentry. Connect estimation and computation strategies to business and finance.

Standard 5.0: Data Analysis and Probability

The student will collect, organize, represent, and interpret data and model situations to determine

theoretical and experimental probabilities.

Learning Expectations:

The student will:

5.1 collect, represent, and describe linear and nonlinear data sets developed from the real world;

5.2 make predictions from a linear data set using a line of best fit;

5.3 interpret a set of data using the appropriate measure of central tendency;

5.4 choose, construct, and analyze appropriate graphical representations for a data set;

5.5 understand the concept of random sampling;

5.6 apply counting principles of permutations and combinations using appropriate technology;

5.7 model situations to determine theoretical and experimental probabilities.

Performance Indicators State :

As documented through state assessment –

At Level 1, the student is able to

- determine the mean (average) of a given set of real-world data (no more than five two-digit numbers);
- interpret bar graphs representing real-world data;
- interpret circle graphs (pie charts) representing real-world data.

At Level 2, the student is able to

- choose the matching linear graph given a set of ordered pairs;
- make a prediction from the graph of a real-world linear data set;
- determine the median for a given set of real-world data (even number of data).

At Level 3, the student is able to

- apply counting principles of permutations or combinations in real-world situations.

Performance Indicators Teacher:

As documented through teacher observation –

At Level 1, the student is able to

- design a strategy for collecting real-world data for a scientific investigation;
- collect and organize real-world data.

At Level 2, the student is able to

- graph real-world data using a variety of representations; debate the selection of a graphical representation which best describes specific data;
- model situations to determine theoretical and experimental probabilities;
- judge the validity of claims made in probabilistic situations;
- defend the sampling method chosen to conduct a survey.

At Level 3, the student is able to

- debate possible conclusions that can be supported by the data;
- make predictions from real-world data using a line of best fit.